

Stability of Z-strings in strong magnetic fields

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Abstract

We show that the Z-strings of the standard electroweak theory can be stabilized by strong external magnetic fields, provided that $\beta^{1/2} \equiv M_H/M_Z \leq 1$, where M_H and M_Z are the Higgs and Z masses. The magnetic fields needed are larger than $\beta^{1/2}B_c$ and smaller than B_c , where $B_c \equiv M_W^2/e$ is the critical magnetic field which causes W-condensation in the usual broken phase vacuum. If such magnetic fields were present after the electroweak transition, they would stabilize strings for a period comparable to the inverse Hubble rate at that time. Pair creation of monopoles and antimonopoles linked by segments of string is briefly considered.

Z -strings are Nielsen-Olesen vortex solutions ‘embedded’ in the electroweak theory [1]. They can be described as flux tubes of the Z gauge boson trapped in a thin core where the Higgs field vanishes. The Higgs develops an expectation value outside the core, where its phase has a net winding around the string. Like the sphaleron [2], Z -strings can carry baryon number and may have played a role in the generation of baryon asymmetry [3]. Nambu’s early work [4] showed that a finite segment of string would be bounded by a magnetic monopole on one end and an antimonopole at the other. Detailed studies of the vortex, by Vachaspati and collaborators [1,5] stirred a new wave of attention. Unlike the vortices in the Abelian Higgs model, which are topologically stable, Z -strings are embedded in a larger theory and their stability depends on the dynamical details. Unfortunately, the analysis showed that, for $\sin^2 \theta_W \approx .23$, Z -strings are unstable [5,17,16]. Then, even if they formed at the phase transition, their lifetime would probably be too short for them to play a cosmological role.

The above studies, however, ignored the effect of a possible background magnetic field. Right after the electroweak phase transition one may expect strong magnetic fields on small scales [6], and it is natural to ask whether these would affect the stability of vortices. The aim of this letter is to show that there is a range for the magnetic field for which Z -strings would be stable. Intuitively, this result can be understood as follows. Recall that the instability of strings is related to the phenomenon of W -condensation [7,8]. The strong Z -flux in the core of the string is coupled to the anomalous magnetic moment of the W -bosons, and this causes the W ’s to condensate. We shall see that by opposing the Z -flux with an external magnetic field, the condensation can be avoided.

To be definite, let us consider the bosonic part of the electroweak energy functional. For static configurations

$$\begin{aligned}
 E = \int d^2x \left\{ - (D_j \Phi)^\dagger D^j \Phi + \lambda \left(|\Phi|^2 - \frac{\eta^2}{2} \right)^2 \right. \\
 + \frac{1}{2} \mathbf{B}_Z^2 + \frac{1}{2} \mathbf{B}_A^2 + \frac{1}{2} G_{ij}^\dagger G^{ij} - i g \mathbf{B}_T \cdot \mathbf{W}^\dagger \times \mathbf{W} \\
 \left. + 2g^2 (\mathbf{W}^\dagger \cdot \mathbf{W})^2 - 2g^2 (\mathbf{W}^\dagger \cdot \mathbf{W}^\dagger)(\mathbf{W} \cdot \mathbf{W}) \right\}. \quad (1)
 \end{aligned}$$

Here $\mathbf{B}_A = \nabla \times \mathbf{A}$, $\mathbf{B}_Z = \nabla \times \mathbf{Z}$, $g\mathbf{B}_T = e\mathbf{B}_A + \gamma\mathbf{B}_Z$, and $G_{ij} = \partial_{[i} W_{j]} - i e W_{[i} A_{j]} -$

$i\gamma W_{[i}Z_{j]}$. As usual, $g = \alpha \cos \theta_W$, $e = g \sin \theta_W$ and $\gamma = g \cos \theta_W$, and we follow the conventions of Ref. [9].

Ambjorn and Olesen have shown [7] that the anomalous mass term

$$-ig\mathbf{B}_T \cdot \mathbf{W}^\dagger \times \mathbf{W} \quad (2)$$

acts like a negative contribution to the W mass squared. An instability develops when the combined field strength $gB_T = |e\mathbf{B}_A + \gamma\mathbf{B}_Z|$ exceeds a certain threshold. For instance, in the usual vacuum (without strings), this happens when the magnetic field contribution exceeds the usual positive W mass term $M_W = g\eta/2$,

$$eB_A > eB_c \equiv M_W^2.$$

Perkins, on the other hand, has pointed out [8] that in the core of a string the field strength \mathbf{B}_Z causes W -condensation for the physical value of $\cos \theta_W$.

It is clear that a constant external magnetic field \mathbf{B}_A antiparallel to \mathbf{B}_Z would reduce the anomalous term (2) in the core of the string, and hence strings might be stabilized in the physical region of parameter space. Of course, detailed analysis is required before we jump to conclusions. If we need magnetic fields larger than B_c to counteract B_Z in the core of the string, these would cause W -condensation outside the core. Also we have to consider all possible modes of fluctuation, not just the formation of W -condensates.

Stability analysis

The Z -string solution is given by

$$\Phi = f(r)e^{in\theta} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad Z = -v(r)d\theta, \quad (3)$$

with all the other fields set to zero. The profiles $f(r)$ and $v(r)$ satisfy the Nielsen-Olesen differential equations [10]. We can always add to the solution a constant magnetic field B_A , with potential

$$A = \frac{B_A}{2}r^2d\theta. \quad (4)$$

This satisfies the field equations without altering $f(r)$ and $v(r)$, since Φ and Z are not magnetically charged. On the other hand, the fluctuations of W and of the upper component of the Higgs field are charged and will feel the presence of B_A .

The stability analysis can be carried out as in the case with no magnetic field [5]. Full details will be reported elsewhere [11]. To check whether the string is a local minimum of (1), one must consider generic perturbations to (3). It can be shown that the energy is minimized when the perturbations to the Z , A , and to the lower component of the Higgs field are set to zero. The reason can be traced to the fact that the abelian Nielsen-Olesen string is stable. The perturbed configuration is then written as

$$\Phi = \begin{pmatrix} \phi_1 \\ f(r)e^{in\theta} \end{pmatrix}; \quad W = W_\mu dx^\mu, \quad (5)$$

with the Z and A fields as given in (3) and (4). The perturbations can be taken to be independent of the z coordinate, and the z components of the gauge fields can be set to zero, since relaxing these conditions always results in an increase of energy [5]. For the same reasons we have set the temporal components of the gauge fields to zero in (1). Generic perturbations can then be expanded in Fourier series

$$\begin{aligned} \phi_1 &= \sum_m e^{im\theta} \chi_m(r), \\ W &= \frac{1}{\sqrt{2}} \sum_l e^{il\theta} [\Theta_l(r) d\theta + iR_l(r) dr], \end{aligned}$$

where χ, Θ and R are arbitrary complex functions of r . Substituting into (1), expanding to quadratic order in the fluctuations and integrating the angular dependence we find that the energy fluctuation is $\delta E = \sum_l E_l$. Here E_l is a quadratic functional depending on χ_{n-l} , Θ_l and R_l . Actually R_l appears only as a Lagrange multiplier. The energy is minimized by imposing $\delta E_l / \delta R_l = 0$, from which R_l can be eliminated. The functional E_l depends now on two functions χ_{n-l} and Θ_l , but by diagonalizing the kinetic term we readily find that there is only one dynamical degree of freedom. Defining the new fields

$$\zeta_l = M \chi_{n-l} + (gf/2) \Theta_l$$

$$\zeta_l^{(g)} = g f r^2 \chi_{n-l} - M \Theta_l,$$

where $M \equiv l + e(B_A/2)r^2 - \gamma v$ and $N \equiv M^2 + (gr^2 f^2/2)$, one finds that $\zeta_l^{(g)}$ drops out. This simplification is due to gauge invariance: it can be shown that $\zeta_l^{(g)}$ corresponds to a pure gauge mode [11,5]. Finally, after lengthy algebra [11]

$$E_l \geq 2\pi \int_0^\infty r dr \zeta_l \mathcal{O} \zeta_l,$$

where

$$\mathcal{O} \equiv -\frac{1}{r} \frac{d}{dr} \left(\frac{r}{N} \frac{d}{dr} \right) + \mathcal{U}(r),$$

$$\begin{aligned} \mathcal{U}(r) &\equiv \frac{f'^2}{N f^2} + \frac{2S}{g^2 r^2 f^2} + \frac{1}{r} \frac{d}{dr} \left(\frac{r f'}{N f} \right) \\ S &\equiv \frac{g^2 f^2}{2} - \frac{M'^2}{N} - r \frac{d}{dr} \left(\frac{M'}{r} \frac{M}{N} \right), \end{aligned}$$

and a prime indicates d/dr .

Just as in the case with no magnetic fields, the problem reduces to finding whether the equation $\mathcal{O} \zeta_l = w \zeta_l$ has any negative eigenvalues w . The boundary conditions of the eigenvalue equation are $\zeta(r \rightarrow 0) \rightarrow r^{|n-l|}$ if $l \neq 0$ and $\zeta(r \rightarrow \infty) \rightarrow 0$ for all l . For $n = 1$ and 2, the $l = 0$ mode is never unstable [11]. For $n \geq 3$ and $l = 0$ the boundary condition at the origin is $\zeta \rightarrow r^n$.

Results for $n=1$

Fig. 1 summarizes the stability of the $l=1$ mode for different values of the magnetic field $B_A = 2 K M_Z^2/\alpha$, in the parameter space $(\beta^{1/2}, \sin^2 \theta_W)$. Here $\beta^{1/2} \equiv M_H/M_Z$, where $M_H = (2\lambda)^{1/2} \eta$ and $M_Z = \alpha \eta/2$ are the Higgs and Z masses. The solid curves are the critical lines separating the regions of stability (right) and instability (left). This instability corresponds to the formation of a W -condensate in the core of the string due to B_Z [8].

For $B_A = 0$ the result coincides with the critical line found in Ref. [5]. As the magnetic field B_A is increased, the critical line moves to the left, so that this mode is stable for

smaller values of $\sin^2 \theta_W$. The interpretation is that the magnetic field opposes B_Z and hence W -condensation is inhibited. In particular, when $K \approx 1$ the $l = 1$ mode is stable for $\sin^2 \theta_W \approx .23$. The vertical lines correspond to $B_A = B_c$. To the right of the vertical lines we have W -condensation in the trivial vacuum, outside the core of the string [7]. Note that for $\beta = 1$ the critical lines $l = 1$ meet with the vertical lines. This is consistent with Ref. [12], where the Bogomol'ny limit ($\beta = 1$) was considered.

However, this is not the whole story, since we have to consider all values of l . When eB_A is larger than γB_Z in the core of the string, W -condensation develops in the direction *opposite* to the one it did when $B_A = 0$. This corresponds to an instability in the mode $l = -1$. In Fig. 2a we plot the critical stability lines for $l = 1$ and $l = -1$ (solid lines) as a function of the applied magnetic field, for the physical value of $\sin^2 \theta_w$. In the region between both lines the strings are (meta-)stable. The dashed line corresponds to $\mathbf{B}_T = 0$ at $r = 0$. Near this line the anomalous mass term almost vanishes inside the core, and hence it is not surprising that we find stability.

We have checked that for $n = 1$ the $l = 0$ mode is stable for all values of B_A . Some critical lines for $|l| > 1$ are also plotted in dot-dashed lines in Fig. 1. They do not restrict the stability region any further.

The experimental lower bound on the Higgs mass gives $\beta^{1/2} \gtrsim .6$. For $.6 \lesssim \beta^{1/2} \leq 1$ the $n = 1$ strings are stable over a range of magnetic fields of relative width

$$\frac{\Delta B_A}{B_A} \approx .2(\beta^{-1/2} - 1). \quad (6)$$

If the magnetic field is frozen in with the plasma, as it is usually assumed [6], we have $B_A \propto a^{-2}(t)$, where $a(t) \propto t^{1/2}$ is the cosmological scale factor. Given a sufficiently large initial magnetic field, this means that strings can be metastable for a time of cosmological order: $\Delta t \approx (\Delta B_A/B_A)t \approx .2(\beta^{-1/2} - 1)t$. This may be sufficient for strings to fall out of thermal equilibrium, which in turn may have consequences for baryogenesis [3].

For $\beta^{1/2} > 1$ and $B_A < B_c$ there are no stable Z -strings. When $B_A > B_c$ the vacuum decays into a lattice of W -condensate vortices [7]. Since the W bosons couple to the Z flux,

we expect that the simple Z-string solution will not exist in this background.

Results for higher winding

Fig. 2b shows the metastable region for a string of winding number $n = 2$. The critical lines for $l = 1$ and -1 are shown as solid lines. The $l = 0$ mode is stable in the whole range, and for $|l| > 1$ the critical lines are less restrictive than the ones depicted. The metastable region, between solid lines, is appreciably narrower than in the case $n = 1$. Like in that case, the region is centered around the locus where $\mathbf{B}_T = 0$ at $r = 0$ (dashed line). Of course the line is not the same as in Fig. 2a, because B_Z is different.

The stable regions for $n = 1$ and $n = 2$ have some overlap, but the case $n = 2$ extends to somewhat lower values of the applied magnetic field. This “overlap” pattern continues for higher n . The loci where the anomalous term vanishes at $r = 0$ are represented in Fig. 3 for different values of n . As n increases, the dashed line moves to lower values of the magnetic field. The stability regions for each n are in the vicinity of these lines. We find [11] that the width of these regions decreases with n , but the stable patch for winding n always has some overlap with the stable patch for winding $n + 1$, which in turn extends to somewhat smaller values of the magnetic field. For $n \rightarrow \infty$ the dashed line tends to the straight line $B_A = \beta^{1/2} B_c$, and the width of the stable region tends to zero. Hence $B_A = \beta^{1/2} B_c$ is the lowest value of the magnetic field for which we can find metastable strings of any winding.

As we cross the critical lines $l = \pm 1$ into the unstable region, the strings may unwind and release their energy in the form of linear excitations on the broken phase vacuum, or settle down into a possible “dressed string” configuration [18,17]. The pattern of stability described above may also suggest that, as magnetic field decreases, the $n = 1$ string becomes unstable and decays into a metastable $n = 2$ string. By decreasing B_A further, this decays into a $n = 3$ string, and so forth. In principle, it is not clear whether this cascade is energetically allowed; $n = 2$ strings are heavier than $n = 1$ strings, but we have to take into account that these strings end in monopoles of twice the charge (in other words, energy can be extracted from the magnetic field). We shall return to this question below.

Pair creation

The magnetic fields needed above have an energy density comparable to the electroweak scale, and one should worry about instabilities of the broken phase in the presence of such fields. As we shall see, for $\beta^{1/2}B_c < B_A < B_c$ the broken phase is only metastable.

It is well known that a magnetic field can pair-produce monopoles and antimonopoles [13]. Electroweak monopoles are linked by Z-strings, so pair production can only happen if the force exerted by the magnetic field on the monopoles exceeds the string tension [14],

$$nQB_A > \mu_n. \quad (7)$$

Here $Q = (4\pi/\alpha) \tan \theta_W$ is the $n = 1$ monopole charge [4], and μ_n is the string tension. For large n the tension is dominated by the constant energy density in the core of the string, and we can neglect gradient terms

$$\mu_n = S_n \left(\frac{\lambda\eta^4}{4} + \frac{B_Z^2}{2} \right).$$

Here S_n is the area of a cross section of the core of the string, and $B_Z = \phi_n/S_n$, where $\phi_n = 4\pi n/\alpha$ is the quantized flux. Minimizing the tension with respect to S_n we find $\mu_n = \beta^{1/2}\pi\eta^2 n$. Comparing to the magnetic force, we conclude that in the large n limit the strings will grow provided that $B_A > \beta^{1/2}B_c$ (larger values of B_A are needed for lower n).

The decay of the metastable phase can proceed through pair production by thermal activation or by quantum tunneling [13,14]. However, the actual decay rate and the “fate” of this “false vacuum” are difficult to assess. The problem is that condition (7) is only satisfied in the regions where the Z-strings of winding n are unstable. As a result, the corresponding instantons will have too many negative modes, and will probably not represent a semiclassical channel of decay [15]. Further work will be needed to clarify this issue.

Finally, we may ask whether a $n = 1$ string can decay into a $n = 2$ string. This process can be thought of as the nucleation of a monopole antimonopole pair on top of an existing $n = 1$ string. For this to be energetically possible, we need $QB_A > \mu_2 - \mu_1$. The transition line $QB_A = \mu_2 - \mu_1$ is depicted in Fig. 2b as a double dot-dashed line. It happens to lie

in the intersection of the regions of metastability for $n = 1$ and $n = 2$. Since the strings involved are both metastable, this process may be mediated by an instanton with just one negative mode, as is required by the general theory of semiclassical tunneling [15].

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FIGURES

FIG. 1. Stability of the $l = 1$ mode for $n = 1$ and different values of the magnetic field $B_A = 2KM_Z^2/\alpha$. The solid curves are critical lines separating the regions of stability (right) and instability (left). The vertical lines correspond to $B_A = B_c$. For $K \approx 1$, the $l = 1$ mode is stable for $\sin^2 \theta_W \approx .23$

FIG. 2. The critical lines of stability for $l = 1$ and $l = -1$ are shown as solid lines, for $n = 1$ and 2, with $\sin^2 \theta_W = .23$. Strings are metastable in the region between solid lines. This region is centered around the locus where $B_T = 0$ at $r = 0$, depicted as a dashed line. For $n = 1$, the critical lines for $l = -2, -3, -4$ are shown in dot-dashed lines. The corresponding instabilities develop to the right of these lines, so they do not restrict the stable region any further. For $n = 2$ the transition line $QB_A = \mu_2 - \mu_1$ is depicted as a double dot-dashed line.

FIG. 3. Locus where $B_T = 0$ at $r = 0$ for some windings. For each n the stability region is in the vicinity of these lines.

Fig. 1

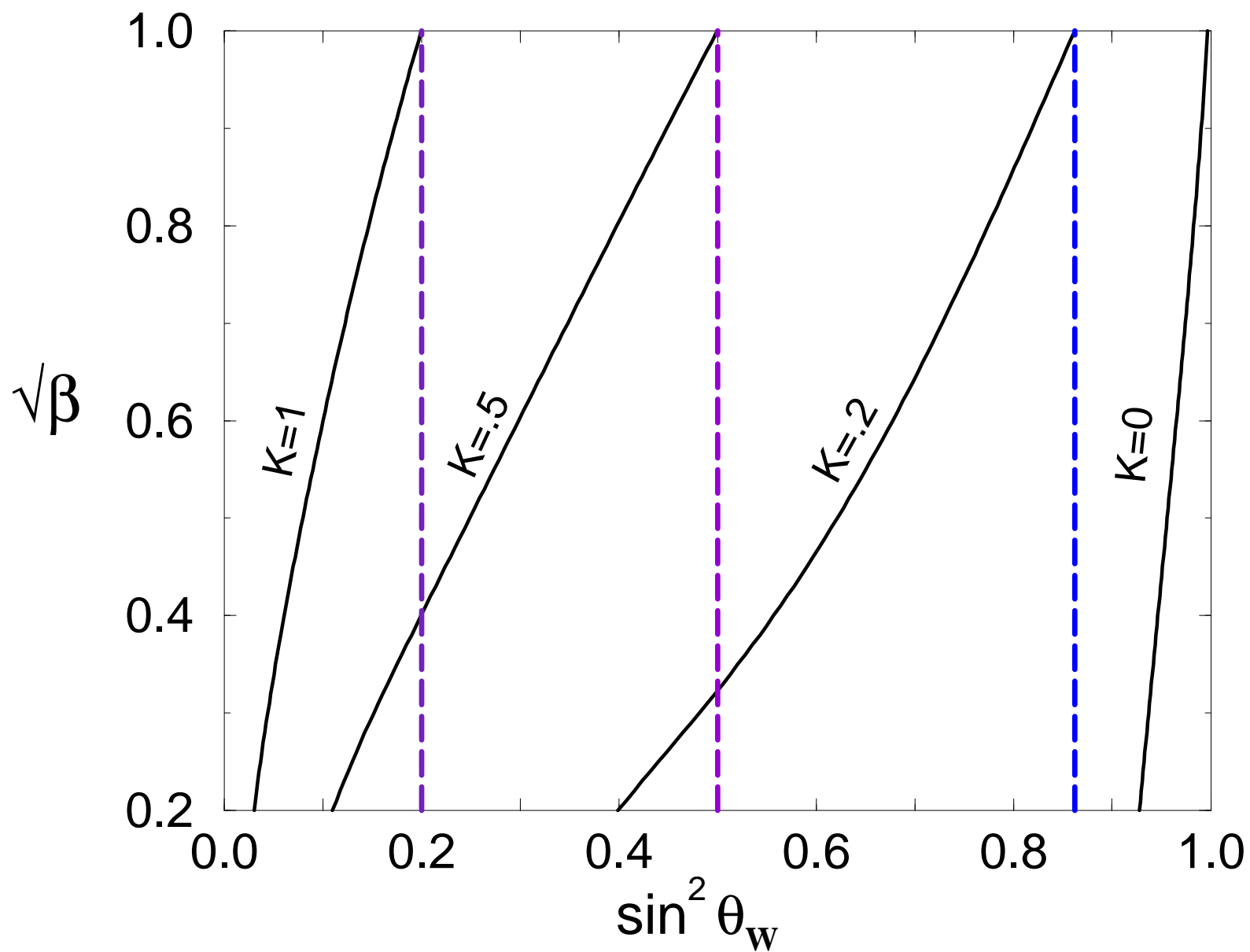


Fig. 2

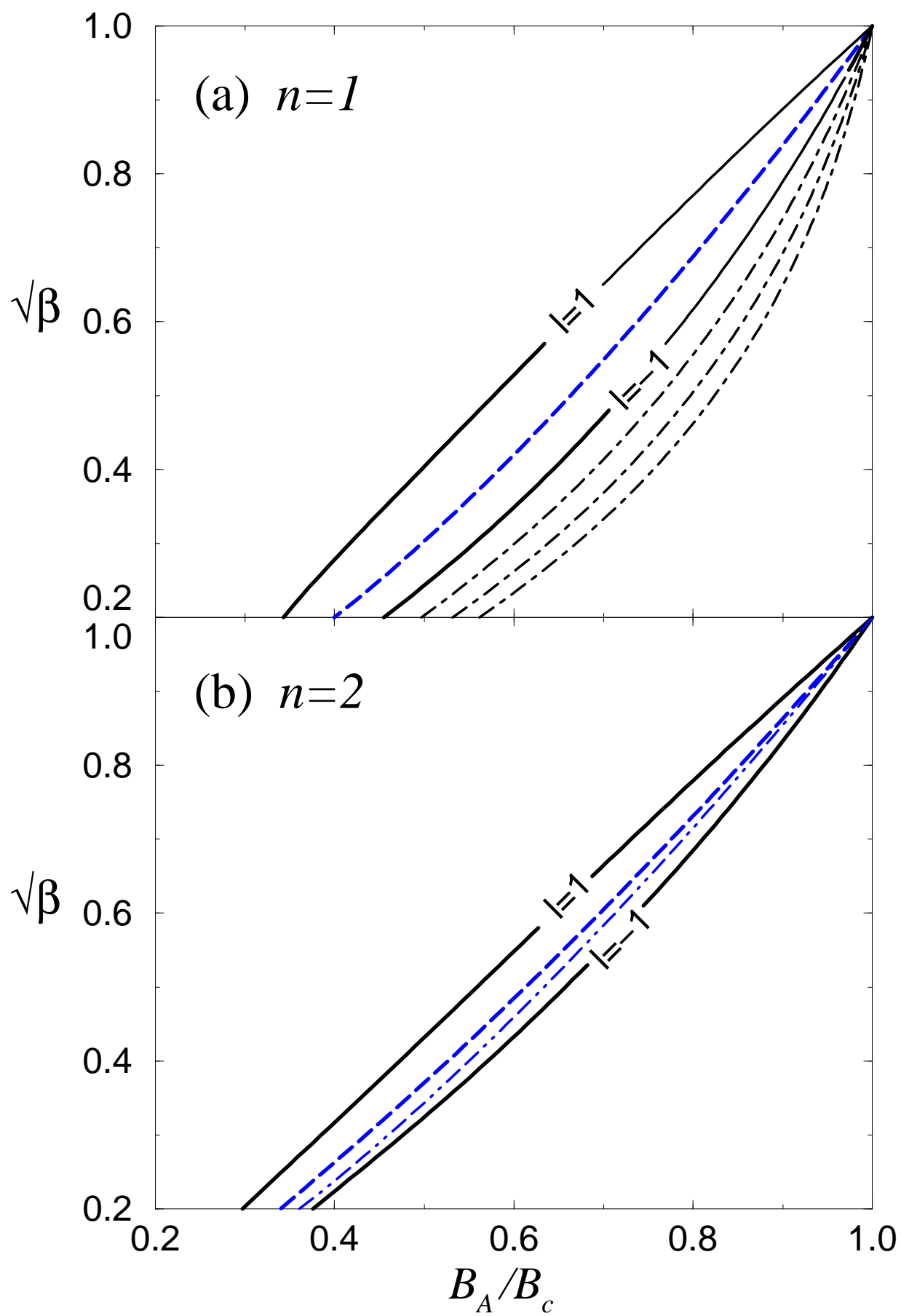


Fig. 3

